AP Calculus Review: Limits

1. Explain what $\lim_{x \to a} f(x) = k$ means.

2. Find the value of k such that the function $f(x) = \begin{cases} x+3 & x \le 2 \\ kx+6 & x < 2 \end{cases}$ is continuous at $x = 2$.

3. True or False:

   a. $\lim_{x \to 0} \frac{|x|}{x} = 1$

   b. If $\lim_{x \to c} f(x) = L$ then $f(c) = L$.

   c. If $f(x) = g(x)$ for all real numbers other than $x = a$, and if $\lim_{x \to a} f(x) = L$, then $\lim_{x \to a} g(x) = L$.

   d. For polynomial functions, the limits from the right and from the left at any point must exist and be equal.

   e. If $f(x)$ is continuous on the interval [0, 1], $f(0) = 2$ and $f(x)$ has no roots in the interval, then $f(x) > 0$ on the entire interval [0, 1].

   f. If $\lim_{x \to a} g(x) = 0$, then $\lim_{x \to a} \frac{f(x)}{g(x)}$ does not exist.

4. Find a rational function having a vertical asymptote at $x = 3$ and a horizontal asymptote at $y = 2$.

5. Draw a graphical counter-example to show the IVT does not hold if $f(x)$ is not continuous in $[a, b]$.

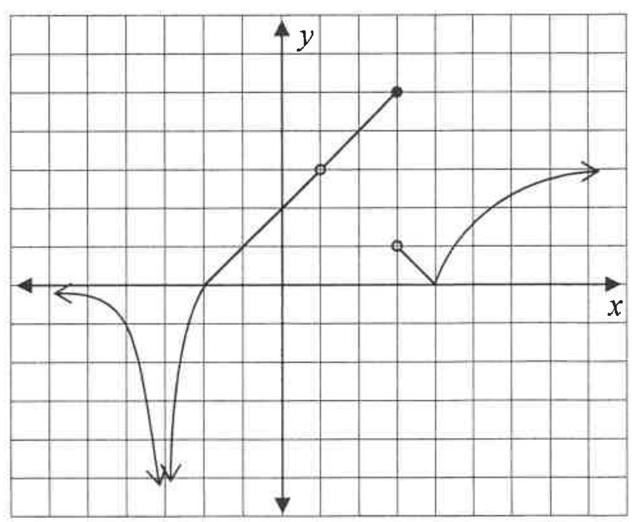
6. Identify all the asymptotes of the graph of the function $y = \frac{2x^2 - 4x}{x^2 - 5x + 6}$.

7. Use the graph of the function $y = f(x)$ at right to evaluate the following limits.

   a. $\lim_{x \to 0} f(x)$ b. $\lim_{x \to 1} f(x)$ c. $\lim_{x \to 3} f(x)$

   d. $\lim_{x \to 4} f(x)$ e. $\lim_{x \to \infty} f(x)$ f. $\lim_{x \to -\infty} f(x)$

   g. $\lim_{x \to -3} f(x)$ h. $\lim_{x \to 3^-} f(x)$ i. $\lim_{x \to 3^+} f(x)$



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8. Evaluate the limits *without* using your calculator:

a. $\lim_{x\to 3}\frac{x^2-3x}{x^2+x-12}$

b. $\lim_{x\to 4}\frac{\sqrt{(x-4)^2}}{x-4}$

c. $\lim_{x\to\infty}\frac{(x^2-3)^2}{2x^3+x^2-6x-3}$

d. $\lim_{x\to\infty} x^4e^{-4x}$

e. $\lim_{x\to\infty}\frac{\ln x}{\sqrt{x}}$

f. $\lim_{x\to\infty}\cos^{-1}\left(\frac{x}{x+1}\right)$

g. $\lim_{x\to\infty} e^{\cos\left(\frac{1}{x}\right)}$

h. $\lim_{x\to 0}\ln|\sin x|$

9. Evaluate $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ for the functions

a. $f(x)=\sqrt{3x+1}$

b. $f(x)=x^3$ (From A2T: $(a+b)^3=a^3+3a^2b+3ab^2+b^3$)

10. Evaluate $\lim_{x\to -\infty}\frac{kx+t}{\sqrt{ax^2+bx+c}}$ assuming $a>0$ (why?).

11. Given that $\lim_{x\to 1}f(x)=36$, $\lim_{x\to 0}g(x)=g(0)=5$, $\lim_{x\to 1}g(x)=-3$, $\lim_{x\to 1}h(x)=0$, evaluate

a. $\lim_{x\to 1}\left(\sqrt{f(x)}-g(x)\right)$

b. $\lim_{x\to 1}\frac{g(x)}{h(x)}$

c. $\lim_{x\to 1}g(h(x))$

12. Use the graphs of the functions f and g at right to evaluate the following limits if they exist.

a. $\lim_{x\to -2}[f(x)+g(x)]$

b. $\lim_{x\to 0}\frac{f(x)}{g(x)}$

c. $\lim_{x\to -3^+}[f(x)+g(x)]$

d. $\lim_{x\to 4}[f(x)g(x)]$

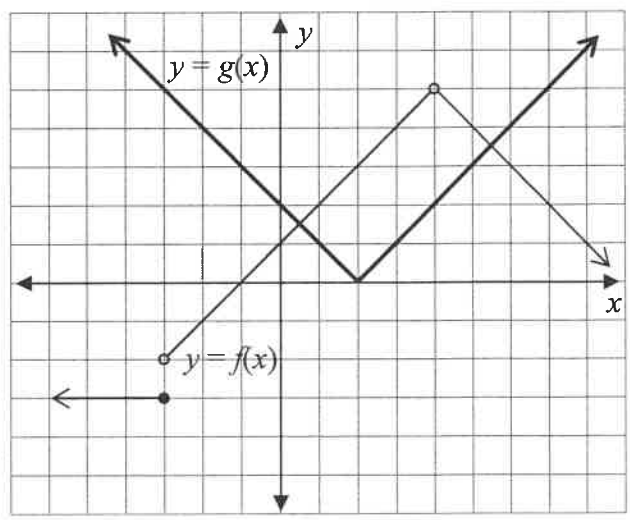
e. $\lim_{x\to 2}\frac{f(x)}{g(x)}$

f. $\lim_{x\to -2}g(f(x))$

g. $\lim_{x\to -\infty}[f(x)+g(x)]$

h. $\lim_{x\to\infty}[f(x)+g(x)]$

i. $\lim_{x\to -\infty}\frac{f(x)}{g(x)}$



13. Kenny didn't worry too much about the test because he was confident he could figure out most limit problems with his calculator. What happened?

AP Calculus HW: Limits – 1

1. a. Explain in your own words what is meant by the statement $\lim_{x \to 3} f(x) = 8$.

   b. Is it possible for the statement to be true if $f(3)$ is undefined? Explain (or illustrate).

   c. Is it possible for the statement to be true if $f(3) = 10$? Explain (or illustrate).

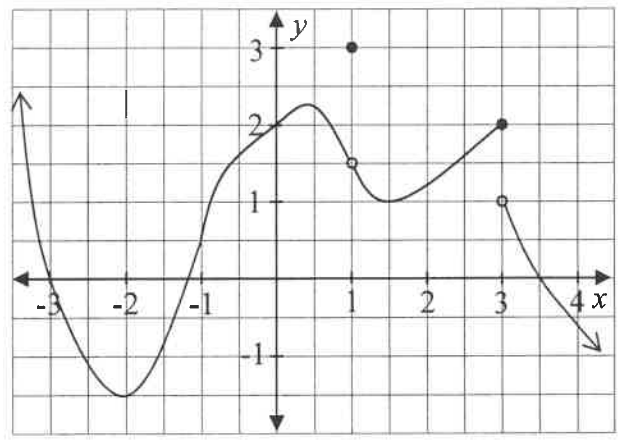
2. a. What is meant by $\lim_{x \to 2^-} f(x) = 6$ and $\lim_{x \to 2^+} f(x) = 4$?

   b. Is $\lim_{x \to 2} f(x)$ defined? Explain.

   c. Is $f(2)$ defined? Explain.

   d. What happens to the function at $x = 2$?

3. Use the graph of f at right to evaluate the following:



   a. $\lim_{x \to 0} f(x)$ b. $\lim_{x \to -2} f(x)$ c. $\lim_{x \to 1} f(x)$

   d. $f(1)$ e. $\lim_{x \to -3} f(x)$ f. $\lim_{x \to 3^-} f(x)$

   g. $\lim_{x \to 3^+} f(x)$ h. $\lim_{x \to 3} f(x)$ i. $f(3)$

4. Sketch a graph of a function that satisfies these conditions: $\lim_{x \to 0^-} f(x) = -1$, $\lim_{x \to 0^+} f(x) = 1$, $\lim_{x \to 2^-} f(x) = 0$, $\lim_{x \to 2^+} f(x) = 1$, $f(0)$ is undefined and $f(2) = 1$.

5. Use your graphing calculator to estimate the value of the following limits. Then *put the answers in your brain.*

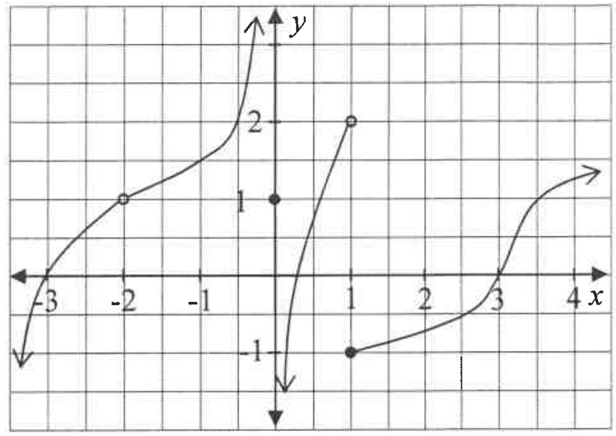
   a. $\lim_{x \to 0} \frac{\sin x}{x}$ b. $\lim_{x \to 0} \frac{1 - \cos x}{x}$ c. $\lim_{x \to 0} \frac{e^x - 1}{x}$

6. Estimate the value of $\lim_{x \to 0^+} (1 + x)^{1/x}$.

7. One night, Kenny was doing his homework. One problem said that $\lim_{x \to 3} g(x) = 0$. Kenny concluded that the function g has a root at $x = 3$. What happened?

AP Calculus HW: Limits – 2

1. Use the graph of f at right to evaluate the following:



   a. $\lim_{x\to -2} f(x)$ b. $\lim_{x\to 1^-} f(x)$ c. $\lim_{x\to 1^+} f(x)$

   d. $\lim_{x\to 1} f(x)$ e. $f(1)$ f. $\lim_{x\to 0} f(x)$

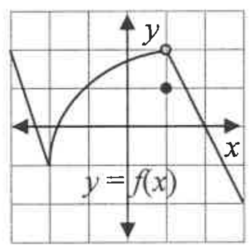
   g. $f(0)$

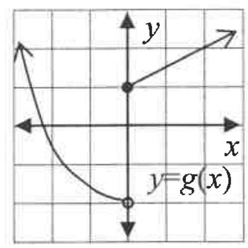
2. Given that $\lim_{x\to 4} f(x)=16$, $\lim_{x\to 4} g(x)=-2$,
   $\lim_{x\to -2} f(x)=f(-2)=7$ (f is continuous at $x=-2$),
   $\lim_{x\to -2} g(x)=0$, $\lim_{x\to 16} f(x)=\sqrt{5}$ and
   $\lim_{x\to 16} g(x)=g(16)=3$ (g is continuous at $x = 16$), evaluate the following limits:

   a. $\lim_{x\to 4}\left[f(x)g(x)\right]$ b. $\lim_{x\to -2}\dfrac{f(x)}{g(x)}$ c. $\lim_{x\to 4}\left[f(x)-g(x)\right]$ d. $\lim_{x\to 4}\sqrt{f(x)}$

   e. $\lim_{x\to 16}\left[f(x)\right]^4$ f. $\lim_{x\to 4}\left[0.5f(x)+4g(x)\right]$ g. $\lim_{x\to 4}(f\circ g)(x)$ h. $\lim_{x\to 4}(g\circ f)(x)$

3. Use the graphs of f and g at right to evaluate the following limits.





   a. $\lim_{x\to 1}\left[f(x)+g(x)\right]$ b. $\lim_{x\to 0}\left[f(x)g(x)\right]$

   c. $\lim_{x\to 2}\dfrac{g(x)}{f(x)}$ d. $\lim_{x\to 0^-}(f\circ g)(x)$

   e. $\lim_{x\to 0^+}(f\circ g)(x)$ f. $\lim_{x\to 2}(g\circ f)(x)$

4. If a function f is *continuous* at $x = a$ (i.e., there is no "break" in the graph of f at $x = a$), then $\lim_{x\to a} f(x)=f(a)$. Evaluating a limit in this way is called "direct substitution." Evaluate the following limits by direct substitution:

   a. $\lim_{x\to 4}\left(2x^2-3x+5\right)$ b. $\lim_{x\to \pi/2}\dfrac{\sin x}{x}$ c. $\lim_{x\to -3}\sqrt{\dfrac{1-x}{x^2}}$ d. $\lim_{x\to 1} xe^x\ln x$

5. a. For $f(x)=\dfrac{x^2-16}{x-4}$, try to evaluate by $\lim_{x\to 4} f(x)$ direct substitution. Then algebraically simplify the function and try again.

   b. For $g(x)=\dfrac{\dfrac{4}{x}-1}{\dfrac{x}{4}-\dfrac{4}{x}}$, try to evaluate by $\lim_{x\to 4} g(x)$ direct substitution. Then algebraically simplify the function and try again.

   c. For $h(x)=\dfrac{x-4}{\sqrt{x}-2}$, try to evaluate by $\lim_{x\to 4} f(x)$ direct substitution. Then rationalize the denominator of the function and try again.

(***This assignment is continued on the next page.***)

d. For $f(x)=\dfrac{x^2-16}{x-4}$, try to evaluate by $\lim_{x\to 4} f(x)$ direct substitution. Then algebraically simplify the function and try again.

e. If direct substitution of a limit gives the form $\dfrac{0}{0}$, does this automatically mean the limit DNE?

f. If direct substitution of a limit gives the form $\dfrac{0}{0}$, will we always be able to "fix" the function to find the limit?

6. Kenny had to evaluate $\lim_{x\to 0}\dfrac{e^{3x}-\cos(2x)}{x}$.

   a. Kenny noticed the denominator goes to 0 and wrote "DNE." What happened?

   b. Given a second chance, Kenny noticed both numerator and denominator go to 0 so he wrote $\dfrac{0}{0}=1$. What happened?

AP Calculus HW: Limits – 3

Evaluate the limits algebraically.

1. $\lim_{x\to 2}\dfrac{x^2-x-2}{x-2}$

2. $\lim_{x\to 0}\dfrac{(x-4)^2-16}{x}$

3. $\lim_{x\to 0}\dfrac{\sqrt{3-x}-\sqrt{3}}{x}$

4. $\lim_{h\to 0}\dfrac{(x+h)^{-1}-x^{-1}}{h}$

5. $\lim_{x\to -3}|x+3|$

6. $\lim_{x\to -3}\dfrac{|x+3|}{x+3}$

7. $\lim_{x\to b}\dfrac{x^4-b^4}{bx-b^2}$

8. Evaluate $\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ for $f(x)=\sqrt{x}$

9. If $f(x)=\begin{cases}1-x & x<0\\ \sqrt{2x} & 0\le x<2,\\ 0.5x^2 & x\ge 2\end{cases}$

This is an example of a *piecewise defined function*. The domain is $(-\infty, \infty)$ but the definition of the function is different on different "pieces" of the domain: linear when $x<0$, part of a radical function for $0\le x<2$ and part of a parabola for $x\ge 2$.

   a. Evaluate the following limits

   (1) $\lim_{x\to 0^+} f(x)$ (2) $\lim_{x\to 0^-} f(x)$ (3) $\lim_{x\to 0} f(x)$ (4) $\lim_{x\to 2} f(x)$ (5) $\lim_{x\to 3} f(x)$

   b. Sketch the graph of f.

10. a. Evaluate $\lim_{x\to a^-}\dfrac{x^2-a^2}{|x-a|}$ b. Evaluate $\lim_{x\to a^+}\dfrac{x^2-a^2}{|x-a|}$ c. What happens to $f(x)=\dfrac{x^2-a^2}{|x-a|}$ at $x=a$?

11. Kenny had to evaluate $\lim_{x\to 0}\dfrac{\sin\frac{\pi}{2}x}{2x}$. Kenny got $\lim_{x\to 0}\dfrac{\sin\frac{\pi}{2}x}{2x}=\lim_{x\to 0}\dfrac{\sin\frac{\pi}{2}}{2}=\dfrac{1}{2}$. What happened?

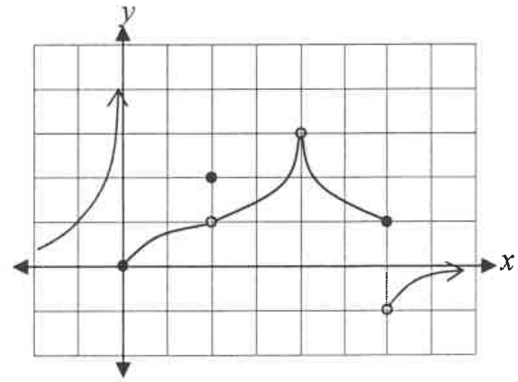
AP Calculus HW: Limits – 4

1. If $\lim_{x\to c} f(x) = L$, which of the following are true?

   a. $\lim_{x\to c^-} f(x) = L$ b. If c is in the domain of f then $f(c) = L$.

   c. f can be made as close as we wish to L (but not necessarily equal to L) by making x close enough to c.

2. Use the graph of f at right to evaluate the following limits.



   a. $\lim_{x\to -1} f(x)$ b. $\lim_{x\to 0} f(x)$ c. $\lim_{x\to 2} f(x)$

   d. $\lim_{x\to 4} f(x)$ e. $\lim_{x\to 0^+} f(x)$ f. $\lim_{x\to \infty} f(x)$

3. Evaluate the following limits:

   a. $\lim_{x\to 2} \cos\left(\frac{\pi}{x+1}\right)$ b. $\lim_{x\to 4} \frac{\sqrt{x}-2}{x^2-16}$ c. $\lim_{x\to 2} \frac{\frac{x^2}{4}-1}{\frac{2}{x}-1}$ d. $\lim_{x\to 1} \cos^{-1}(\ln x)$

4. Evaluate $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ for $f(x) = \frac{1}{x}$.

5. Let f be the function $f(x) = \frac{x^2-1}{x-1}$.

   a. Evaluate $f(1)$. b. Evaluate $\lim_{x\to 1} f(x)$. c. Sketch the graph of f.

   d. Explain why f is not continuous at $x = 1$. (Do not simply say there is a "break" in the graph there; explain *why* there is a break in the graph.

6. Let f be the function $f(x) = \begin{cases} \frac{x-1}{|x-1|} & x \neq 1 \\ 1 & x = 1 \end{cases}$.

   a. Evaluate $f(1)$.
   b. Evaluate $\lim_{x\to 1} f(x)$.
   c. Sketch the graph of f.
   d. Explain why f is not continuous at $x = 1$.

7. Let f be the function $f(x) = \begin{cases} x^2-1 & x \neq 1 \\ 1 & x = 1 \end{cases}$.

   a. Evaluate $f(1)$.
   b. Evaluate $\lim_{x\to 1} f(x)$.
   c. Sketch the graph of f.
   d. Explain why f is not continuous at $x = 1$.

8. Make a hypothesis based on the previous three problems: What conditions must be met for a function to be continuous at some point $x = c$?

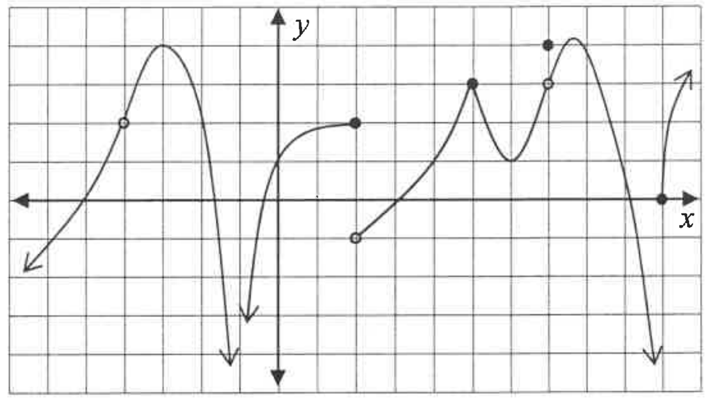
9. Kenny had to evaluate $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ for $f(x) = x^2$. He got $\lim_{h\to 0} \frac{x^2+h-x^2}{h} = \lim_{h\to 0} \frac{h}{h} = 1$. What happened?

AP Calculus HW: Limits – 5

1. Write the requirements for a function g to be continuous at $x = k$.

2. Sketch the graph of a function that has a jump discontinuity at $x = -2$, a removable discontinuity at $x = 1$ and an infinite discontinuity at $x = 4$.

3. The graph of f is shown at right. From the graph, name the x–values at which f is discontinuous. For each, tell what type of discontinuity it is.



4. An airport parking lot charges \$2 for the first half hour or part thereof and \$1 for each additional half hour or part thereof. Sketch the graph of the parking charge as a function of time parked and explain the *real–life* implications of the discontinuities in the graph (in other words, what do the discontinuities mean to the person who parks her car there?)

5. For each function, name the x-value(s) where the function has a discontinuity, tell what type of discontinuity it is and, if the discontinuity is removable, tell how to remove it.

   a. $f(x) = \dfrac{x^2 - 9}{x - 3}$

   b. $g(x) = \dfrac{x + 1}{x^2 - 4x + 4}$

   c. $f(x) = \begin{cases} \sqrt{4 - x^2} & x < 0 \\ 2 - x^2 & 0 \le x \le 2 \\ x - 3 & x > 2 \end{cases}$

6. Find the value of a for which the function $f(x) = \begin{cases} \dfrac{x - a}{x^2} & x < 2 \\ ax + 5 & x \ge 2 \end{cases}$ will be continuous at $x = 2$.

7. Find the values of a and b for which the function $f(x) = \begin{cases} \dfrac{x^2 - ax}{2x - 8} & x < 4 \\ 8 - bx & x \ge 4 \end{cases}$ will be continuous at $x = 4$.

8. Kenny was given a continuous function g and told that $\lim\limits_{x \to 5} g(x) = -2$ and asked to evaluate $g(5)$. Kenny, "learning" from a previous mistake, said there was not enough information. What happened?

AP Calculus HW: Limits – 6

1. Does the IVT apply in each case? If the theorem applies, find the guaranteed value of c. Otherwise, explain why the theorem does not apply.
   a. $f(x) = x^2 - 4x + 1$ on the interval [3, 7], $N = 10$.
   b. $f(x) = \sqrt{2x-4}$ on the interval [2, 10], $N = 5$.
   c. $f(x) = \dfrac{4x-3}{x-3}$ on [0, 6], $N = 4$

2. The table below shows selected values of a function f that is continuous on [2, 9].

x	2	3	4	5	6	7	8	9
$f(x)$	–1	0	3	1	–2	–5	–3	4

   a. What is the least number of roots f may have in the interval [2, 9]? Justify your answer.
   b. Would the answer be the same if f were not continuous? Explain.

3. The function f is continuous on the closed interval [–1, 1] and has values that are given in the table at right. For what values of k will the equation $f(x) = 2$ have at least two solutions in the interval [–1, 1] ?

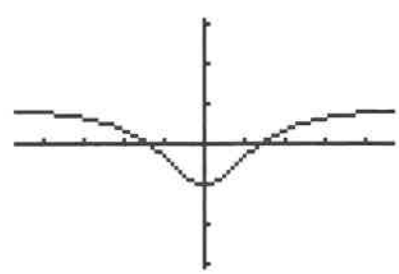
x	–1	0	1
$f(x)$	3	k	5

4. A function g has domain [2, 5] with $g(2) = 6$ and $g(5) = -1$. Which of the following is true?
   (A) g *must* have a root in [2, 5].
   (B) g *may* have a root in [2, 5].
   (C) g *can not* have a root in [2, 5].

5. Suppose a function f is continuous on the interval [1, 5] except at $x = 3$ and $f(1) = 2$ and $f(5) = 7$. Let $N = 4$. Draw two possible graphs of f, one that satisfies the conclusion of the IVT and another that does not satisfy the conclusion of the IVT.

6. a. Suppose N and D are positive numbers. What happens to the value of $\dfrac{N}{D}$ if N is constant and $D \to 0$?
   b. Suppose N is a positive number. Evaluate the following limits. When possible, be more specific than just DNE.
   (1) $\displaystyle\lim_{x\to a^+} \frac{N}{x-a}$ (2) $\displaystyle\lim_{x\to a^-} \frac{N}{x-a}$ (3) $\displaystyle\lim_{x\to a} \frac{N}{x-a}$
   (4) $\displaystyle\lim_{x\to a^+} \frac{N}{(x-a)^2}$ (5) $\displaystyle\lim_{x\to a} \frac{N}{(x-a)^2}$ (6) $\displaystyle\lim_{x\to a} \frac{N}{(x-a)^2}$
   c. Based on (1) – (3) above, sketch the behavior of the graph of $f(x) = \dfrac{N}{x-a}$ near $x = a$.
   d. Based on (4) – (6) above, sketch the behavior of the graph of $f(x) = \dfrac{N}{(x-a)^2}$ near $x = a$.
   e. How would the graphs in part *d* change if N were a negative number?

8. Kenny graphed the function $g(x) = \dfrac{(x^2-2)^2}{x^4-4}$. He could see that $f(1) < 0$ and $f(2) > 0$ so by the IVT, Kenny concluded there is a root in (1, 2). What happened?



AP Calculus HW: Limits – 7

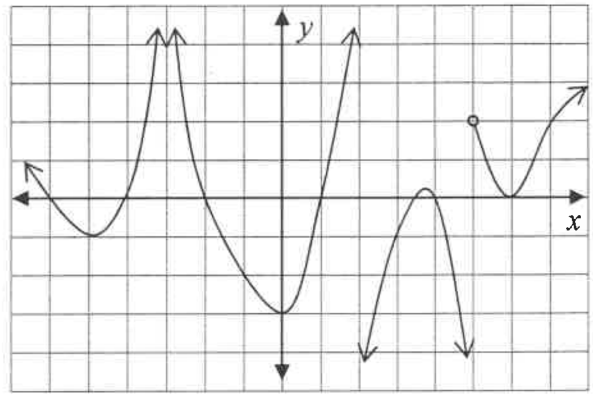
1. Use the graph of f at right to answer the following:

   a. $\lim_{x\to -3} f(x) =$ b. $\lim_{x\to 2^-} f(x) =$

   c. $\lim_{x\to 2^+} f(x) =$ d. $\lim_{x\to 5^-} f(x) =$

   e. $\lim_{x\to 5^+} f(x) =$

   f. Write equations for all the vertical asymptotes of f.



2. What is the difference between the statements $\lim_{x\to 3} f(x) = \infty$ and $\lim_{x\to 3^+} f(x) = \infty$?

3. Can the graph of a function f intersect a vertical asymptote? Illustrate with a graph.

Evaluate the limits:

4. $\lim_{x\to 2} \dfrac{1}{(x-2)^4}$ 5. $\lim_{x\to -2^+} \dfrac{x-2}{x^2(x+2)}$ 6. $\lim_{x\to \frac{\pi}{2}^-} \sec x$ 7. $\lim_{x\to 2} \ln|x-2|$

8. According to Einstein's theory of relativity, the mass of a particle with velocity v is given by

   $m(v) = \dfrac{m_o}{\sqrt{1-\dfrac{v^2}{c^2}}}$ where c is the speed of light.

   a. What does m_0 represent?

   b. What happens to m as $v \to c^-$?

9. a. Use your calculator to help evaluate each of the following limits.

   (1) $\lim_{x\to\infty} \dfrac{6x-8}{3x^2-2x-4}$ (2) $\lim_{x\to\infty} \dfrac{6x^2-8}{3x^2-2x-4}$ (3) $\lim_{x\to\infty} \dfrac{6x^3-8}{3x^2-2x-4}$

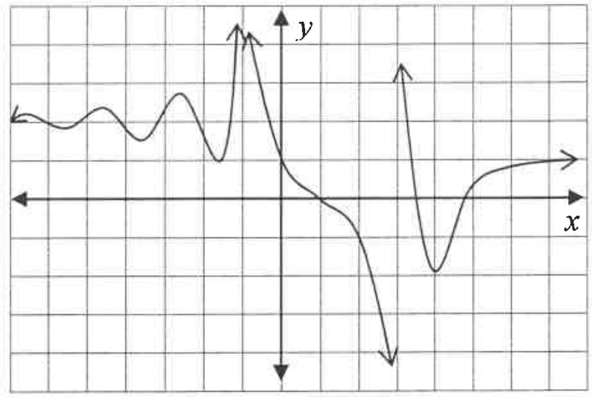
   b. Based on (1) – (3) above, hypothesize a general rule for $\lim_{x\to\infty} \dfrac{p(x)}{q(x)}$ where p and q are polynomials with leading coefficients of P and Q respectively and

   (1) Degree $p <$ degree of p (2) Degree of $p =$ degree of q (3) Degree of $p >$ degree of q

10. Kenny was asked to find all the asymptotes of the function $g(x) = \dfrac{x+\cos(\pi x)}{x-1}$. He wrote $x = 1$ and $y = 1$. What happened?

AP Calculus HW: Limits – 8

1. The graph of f at right has four asymptotes.
   a. $\lim_{x\to\infty} f(x) =$ b. $\lim_{x\to-\infty} f(x) =$
   c. $\lim_{x\to-1} f(x) =$ d. $\lim_{x\to 3^-} f(x) =$
   e. Write the equations of all the asymptotes of f.



2. Draw sketches to illustrate the difference between $\lim_{x\to 3} f(x) = \infty$ and $\lim_{x\to\infty} f(x) = 3$.

3. Can a function intersect a horizontal asymptote? Illustrate with a graph.

4. How many different horizontal asymptotes can one function have? Illustrate with a graph.

5. Evaluate $\lim_{x\to\left(\frac{\pi}{2}\right)^+} e^{\tan x}$

6. Evaluate both limits assuming $a > 0$: $\lim_{x\to\infty} \frac{\sqrt{a^2x^2+1}}{bx+c}$ and $\lim_{x\to-\infty} \frac{\sqrt{a^2x^2+1}}{bx+c}$

Find the indicated limit. Do not use your calculator.

7. $\lim_{x\to\infty} \frac{6x^3-4x^2+2x-7}{2x^3-16}$

8. $\lim_{x\to-\infty} \frac{200x^3}{x^4-200x^3}$

9. $\lim_{x\to\infty} \frac{12x-4x^2}{2x^2-3x-5}$

10. $\lim_{x\to\infty} \frac{x^2-9}{(2x+1)^2}$

11. $\lim_{x\to\infty} \frac{\sqrt{x^4+25}}{1-x^2}$

12. $\lim_{x\to-\infty} \frac{4x-8}{\sqrt{x^2+16}}$

13. $\lim_{x\to\infty} \sin x$

14. $\lim_{x\to\infty} \frac{\sin x}{x}$

15. $\lim_{x\to\infty} \frac{\ln x}{x}$

16. $\lim_{x\to\infty} \frac{e^x}{x^{12}}$

17. $\lim_{x\to\infty} e^{-x}\cos x$

18. $\lim_{x\to-\infty} xe^x$

19. $\lim_{x\to\infty} \cos\left(e^{-x}\right)$

20. $\lim_{x\to\infty} x^2e^{-\sin x}$

21. $\lim_{x\to\infty} x^ne^{-x}$

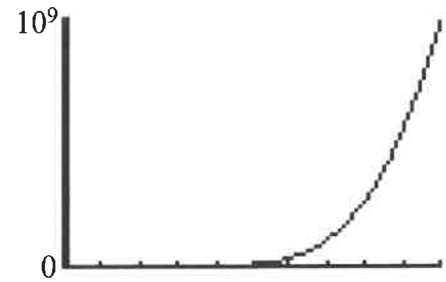
22. $\lim_{x\to\infty} \frac{\ln x}{x^{1/3}}$

23. Suppose $P(x)$ and $Q(x)$ are polynomial functions with leading coefficients 3 and 2 respectively. Evaluate $\lim_{x\to\infty} \frac{P(x)}{Q(x)}$ if
   a. The degree of P is less than the degree of Q.
   b. The degree of P is equal to the degree of Q.
   c. The degree of P is greater than the degree of Q.

24. A raindrop forms in the atmosphere and begins to fall to earth. If we assume air resistance is proportional to the speed of the raindrop, then the drop's velocity as a function of time is given by $v(t) = \frac{mg}{k}\left(1-e^{-kt/m}\right)$ where m is the mass of the raindrop, g is acceleration of gravity and k is a positive constant.
   a. Find $\lim_{t\to\infty} v(t)$. The answer is called the "terminal velocity" of the raindrop.
   b. Sketch the graph of $v(t)$.

25. Suppose f is a function such that $\dfrac{6x}{2x+1} \le f(x) \le 3\left(1-e^{-x/2}\right)$ for all $x > 5$. Find $\lim\limits_{x\to\infty} f(x)$ and justify your answer.

26. Kenny had to evaluate $\lim\limits_{x\to\infty} \dfrac{x^{12}}{2^x}$.



a. Kenny made the graph at right and concluded the limit is infinite. What happened?

b. Given a second chance, Kenny noticed both numerator and denominator go to ∞ so he wrote $\dfrac{\infty}{\infty} = 1$. What happened?

AP Calculus HW: Limits – 9

1. Evaluate the following limits:

a. $\lim_{x\to\infty} \dfrac{x^2-4x-5}{2x^2-1}$ b. $\lim_{x\to\infty} \dfrac{e^{\cos x}}{\sqrt{x}}$ c. $\lim_{x\to 0} \tan^{-1}\left(\dfrac{1}{x^2}\right)$ d. $\lim_{x\to\infty} x^6 e^{-x/6}$ e. $\lim_{x\to\infty} x^2 e^{-x} \ln x$

2. (No calculator.) Where does the function $f(x)=\dfrac{x^2+2bx+b^2}{x^2-b^2}$ have a vertical asymptote?

(A) At $x=-b$ only (B) At $x=b$ only (C) At $x=-b$ and $x=b$ (D) Nowhere

3. (No calculator.) The function $f(x)=\dfrac{\sqrt{x^2-1}}{x-1}$ has how many

a. roots? b. vertical asymptotes? c. horizontal asymptotes?

4. Find the values of a and b that will make $f(x)=\begin{cases} x^2 & x\le 2 \\ ax+b & 2<x\le 4 \\ \sqrt{x-3} & x>4 \end{cases}$ continuous for all x.

5. Suppose f is continuous on [–2, 5]. $f(-2)=6$, $f(3)=-4$ and $f(5)=-1$. Which of the following are true? Justify your answers.
a. f has a root in the interval [–2, 3].
b. f has no root in the interval [3, 5].
c. $\lim_{x\to 2} f(x)=f(2)$
d. The equation $f(x)=-2$ has at least two solutions in [–2, 5].
e. $\lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)$.
f. f has an absolute maximum value in [–2, 5].
g. f has an absolute maximum value in (–2, 5).
h. There is at least one solution to $f(x)=-1$ in (–2, 5).
i. There is at least one solution to $f(x)=6$ in (–2, 5).

6. Evaluate $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ for $f(x)=x^2-3x+1$.

7. Kenny was to evaluate $\lim_{x\to 0} x\cot x$. He reasoned $\lim_{x\to 0} x\cot x=\left(\lim_{x\to 0} x\right)\left(\lim_{x\to 0}\cot x\right)=(0)(\text{Whatever})=0$.

What happened?